12/3/2011 Trigonometry Periods 6 & 7 Bruce Brant / Inna Diordieva

Lesson Structure	Instructions	Time Allotted for this Section 120 min.
A. Climate change topic	*Why is it Hotter at the Equator? The tropics are one of the defining features of the Earth's climate system. Climate change effect - the expansion of the tropical band.*	
B. Relevant mathematical topic being addressed	*Right Triangle Trigonometry* *Use TI 83/84 graphing calculator for curve fitting, data modeling.*	
C. Technological tool/software that will be used for data analysis	*TI 83/84 graphing calculator*	
C. California math framework standard/s being addressed:	*Trigonometry* **12.0** Students use trigonometry to determine unknown sides or angles in right triangles. Reinforcement of *Algebra and Functions* standards from previous years *Mathematical Reasoning* **2.0** Students use strategies, skills, and concepts in finding solutions	
D. Topic Objectives:	At the end of a 120-minute session, TSWBAT: 1. *Solve right triangle using trigonometric functions:* *1.1 Demonstrate that triangle sides could be found through the use of trigonometric functions.* *1.2 Demonstrate that in a right triangle if one angle and one side are known, then the other angle and the other two sides can be solved for easily.* 2. *Apply trigonometry in real life problems solving.* 3. *Explain why it is hotter in the tropics and how global warming affects the climate zones' distribution.*	

E. Instructional materials (worksheets/ activities) needed to accomplish the whole lesson	**Materials for mathematics activity** • Worksheet 1 (see Attachment 1) – Calculation of angle of incidence of the sunlight • Worksheet 3 (see Attachment 3) – Data analysis • Visuals (see Attachment 4) • Assessment worksheet 4 (see Attachment 5) • TI 83/84 calculator **Materials for science activity** • 5 identical Celsius thermometers per group of students (note: *thermometers need to be alcohol-filled*, backed with glass or metal) • Reflector lamp with clamp and 60-watt light bulb • Ring stand with iron ring • Utility clamp (used to secure the ring stand) • Prepared 5 envelops from black construction paper per each group of students) • Several books to prop thermometers, protractor • Worksheet 2 (see Attachment 2) – Science activity • Digital timer	
F. Opening Activity/Purpose	*Measuring Shadow length. 1st try* *Students are going outside, working in groups of 5, and measuring shadow length of one of the students, measure height of this student. Returning to the class and calculating angle of incidence of sunlight using Worksheet 1. (Connecting to previous lesson topic – trigonometric functions - to the use in right triangle). Teacher observes groups and assists those that are struggling with questions like:* *a. How do we define tan θ through the ratio of right triangle sides?* *b. The angle of incidence of sunlight is the angle between a ray incident on a surface and the line perpendicular to the surface at the point of incidence (called the normal). What is the connection between our angle θ and the angle of incidence?*	*15 min.*
G. Main Activity	*<u>Introduction lecture:</u> We found the angle of the incidence of sunlight using trigonometric functions. We will repeat our measurement in an hour to see if angle of the angle changes. The angle we are considering in this activity depends on the Earth rotation (from West to East). Now we are going to look at another direction (South-North) and investigate if there is the different heating effects of sunlight, and if it is, how the angle of direct sunlight affects heat differential.* *Questions to engage students: Do you expect your shadow to*	*10 min.*

	be the same on the second try? Does anybody notice that it is colder in the morning compared to the day time?	
	Science activity. Description is on Worksheet 2. Class goes to science lab where science teacher assists with the experiment. Students enter experiment data into table in Analysis Worksheet 3. Then they plot time lines for each of the thermometers on the grid provided. *Students pick up and store their equipment properly and then the science teacher leads a brief discussion to assist them filling out the activity questions in the worksheet.*	*30 min.*
	Analyzing data. Students enter the final column of data from worksheet 2 into the TI 83/84 calculator to see what the "soaked" temperature data looks like as a function of incidence angle. Instructions: *Set plotting mode to "FUNC" in MODE menu* *Enter the incidence angle data into L1, "{0,30,45,60,90} STO L1"* *Enter the temperature data into L2, "{T1,T2,T3,T4,T5} STO L2"* *Initialize STAT PLOT to use L1 and L2 by going to the STAT/SetUpEditor and pressing enter* *Clear the Y= editor* *Set STAT PLOT 1 to line mode and make sure it's set to use L1 and L2* *Press ZOOM 9 to plot the data* *In the STAT/CALC function, select LinReg to run the linear regression function, then GRAPH it and observe the fit.* *TI 83/84 calculator.* *Students are to attempt at least one of the other curve fit options, compare the results to the linear fit (including looking at the residuals) and make a recommendation for modeling in worksheet 2.*	*15 min.*
	Break	*10 min.*
	Measuring Shadow length. 2nd try. Class goes back outside and takes second shadow measurement to complete the table in Worksheet 1. Teacher asks students if they can solve the right triangle using one side and one non-right angle. Students now complete the second table to practice their knowledge of trig functions. Students return to the math classroom.	*10 min.*
	Impact of latitude lecture. Using Visuals (see Attachment 4) ask student to give their hypothesis of the cause of the	*10 min.*

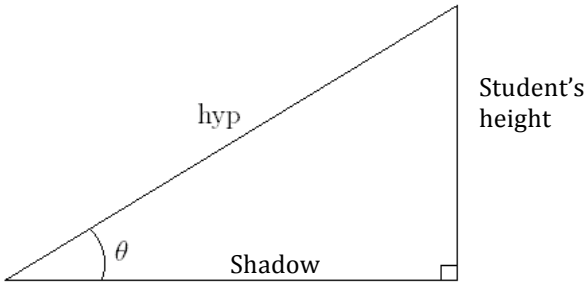
	different heating effect. Help them recognize that the solar radiation passing through the line (b) perpendicular to its origin (the sun) is constant. However, the length of the Earth being heated by this energy varies depending on the latitude (e.g., AB vs. DE). To distribute this constant energy over the linear unit of Earth surface, AB, we have to divide the total amount of the energy by the length of the segment. Teacher uses the following guiding questions to lead students to the answers to questions 1 through 3 in worksheet 4. *a. How can we relate the length of AB to angle α and AC?* *b. What is the relationship between the angle of incidence (α) and the Earth's latitude (φ)?* *c. Can we prove that AB would be always longer than AC?* *Teacher congratulates students for using trigonometry to explain this natural phenomenon.*	
H. Closing Activity	<u>Whole class discussion</u>. *a. What is happening with the cosine when the angle increases from 0° towards 90° (not greater than 90°)?* *b. If we multiply the constant solar energy by a value less than 1 (e.g, cos (α)), what can we say about the result? Would it be smaller or bigger than the total energy?* *c. What is happening with the area illuminated by the solar radiation as this angle increases?* *d. How does the amount of the radiation received by area change?* *e. Based on your answers to b and c, how does the temperature change as you increase the angle of incidence?* *f. If you were given a data table that listed the average yearly temperatures for cities as you travel from the equator towards a Pole, what trend do you think you would see in the temperatures? If a trend exists, what should this trend be and why would it exist?* <u>Global warming issues discussion</u>. Scientists have observed changes in Earth's temperatures in recent years which are changing the distribution of temperatures as well.	*15 min.*

	The climatologists define the tropical boundaries in a more complicated manner, based on different sets of criteria, which are mostly connected to the way the air and oceans circulate around the hot equatorial region. Directly over the equator, the hot air rises, bringing with it moisture that accounts for tropical storms. Further away from the equator, the air descends, which tends to make these subtropical regions drier. *g. How it affects the environment, the animals' habitats?* *h. Do you think that the expansion of the tropical band would affect the global climate system?* *Climate change is having a dramatic impact on the tropics by pushing their boundaries towards the poles. This expansion has potentially important implications for subtropical societies and may lead to profound changes to the global climate system. The movement of large-scale atmospheric circulation systems towards the poles (such as jet streams and storm tracks) could result in shifts in precipitation patterns affecting natural ecosystems, agriculture and water resources. The expansion of the tropical band could exacerbate global warming by increasing the rate at which water vapor an important greenhouse gas is being pumped naturally into the upper atmosphere.*	
I. Assessment	*Students work in their groups to complete Assessment Worksheet 4 (questions 4-7)*	*5 min.*

Attachment 1

Worksheet 1

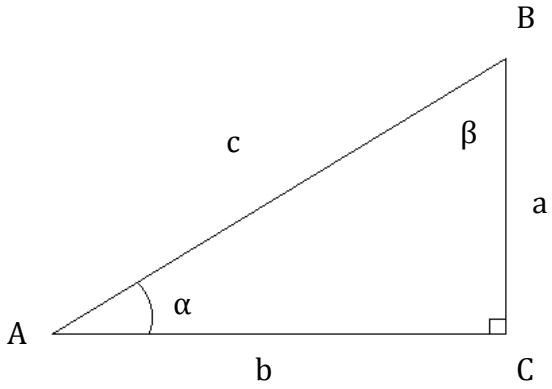
Calculation of angle of incidence of the sunlight



Try	Time	Height	Length of the shadow	Trigonometric function ____ value	Angle θ	Angle of incidence
1						
2						

Express

c through a and β	
c through a and α	
c through b and β	
c through b and α	
a through c and β	
a through c and α	
a through b and β	
a through b and α	
b through a and β	
b through c and β	
b through c and α	



b through a and α	

Attachment 2

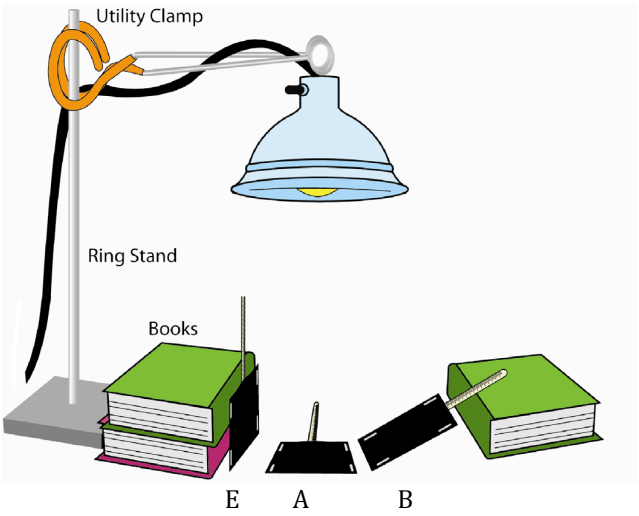
Worksheet 2

Why is it Hotter at the Equator?

Objective
The student will investigate the different heating effects of sunlight as evidenced by observation, documentation, and explanation of how the angle of direct sunlight affects heat differential.

Procedure

1. Attach the lamp to a ring stand and ensure it will not move during the experiment. Use the utility clamp to secure it to the table. Adjust the lamp on the stand so that the light bulb is centered approximately 10 centimeters above the bulbs of the thermometers.
2. Before turning on the lamp, record the temperature of all five thermometers on the student worksheet under the "0 minutes" column of the data table.
3. Place one thermometer (A) horizontally, place three other thermometers (B, C, and D) at angles 30°, 45°, and 60° correspondingly (thermometers C and D are not shown on the picture) using protractor to ensure the correct angle. Place the last thermometer € vertically.
4. Turn on the lamp and record the temperatures for each thermometer every 3 minutes for 15 minutes (a total 6 samples). Do not move the thermometers when reading the temperatures. Record all temperatures on the data table on the student worksheet.
5. Using the data collected, create a graph and plot the results under time versus temperature for all five thermometers using different lines (different colors, solid, dashed, etc.) to show the results of each thermometer.



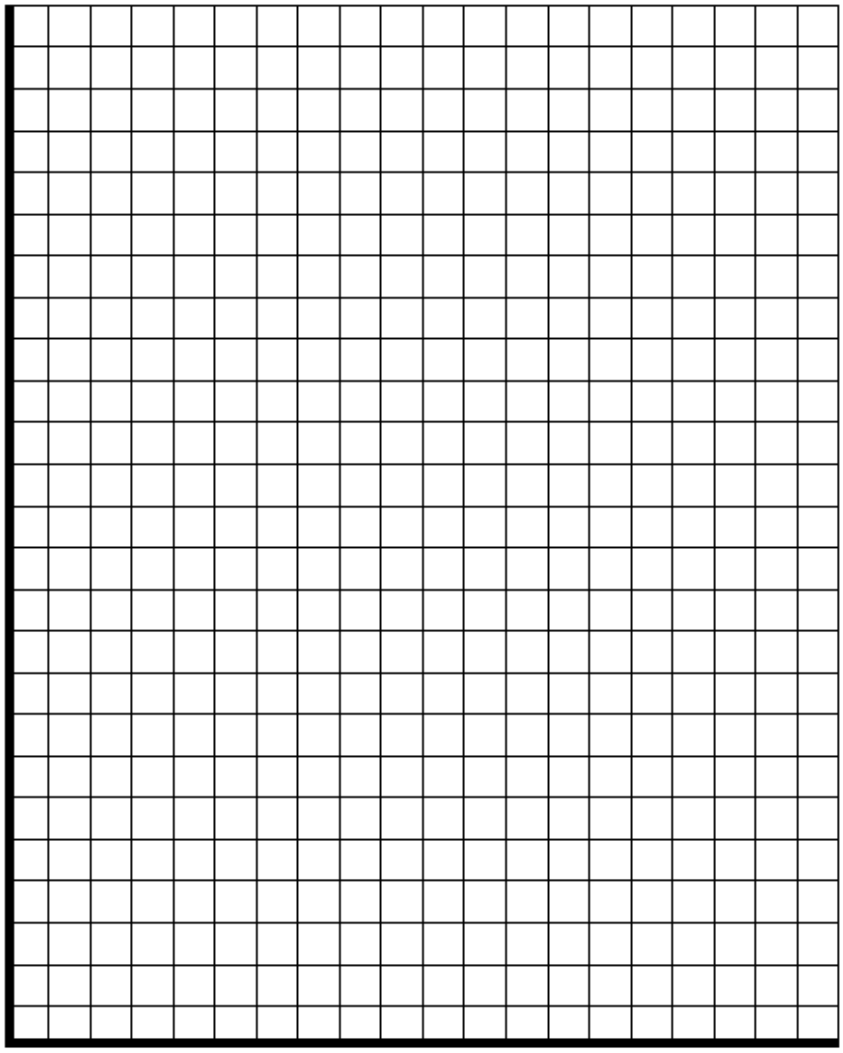
Attachment 3

Data Analysis

Temperature Differentials (due to angle of thermometer)

	0 minutes	3 minutes	6 minutes	9 minutes	12 minutes	15 minutes
Thermometer A						
Thermometer B						
Thermometer C						
Thermometer D						
Thermometer E						

Graph the data from the Temperature Differentials Table. Include: Title, labels for each axis, scale, and key.



Which thermometer showed the greatest temperature increase? Why do you think this one had the greatest increase?_____

Which thermometer(s) best represents the way sunlight strikes the equator? _____

Which thermometer(s) best represents the poles? _____

How good was your linear fit to the soaked temperature data?_____

_____

What other curve fit to the data did you explore and how would you categorize that fit?

_____

For two of the methods you used to try to fit the data on the graphing calculator, please list the resulting equation and the residuals. Identify the maximum residual for both cases

Method 1 _____ Method 2 _____

Equation 1	_____	Equation 2	_____
Residuals:			
0^0	_____		_____
30^0	_____		_____
45^0	_____		_____
60^0	_____		_____
90^0	_____		_____

Based on your analysis, what equation do you recommend to model soak temperature as a function of incidence angle? _____

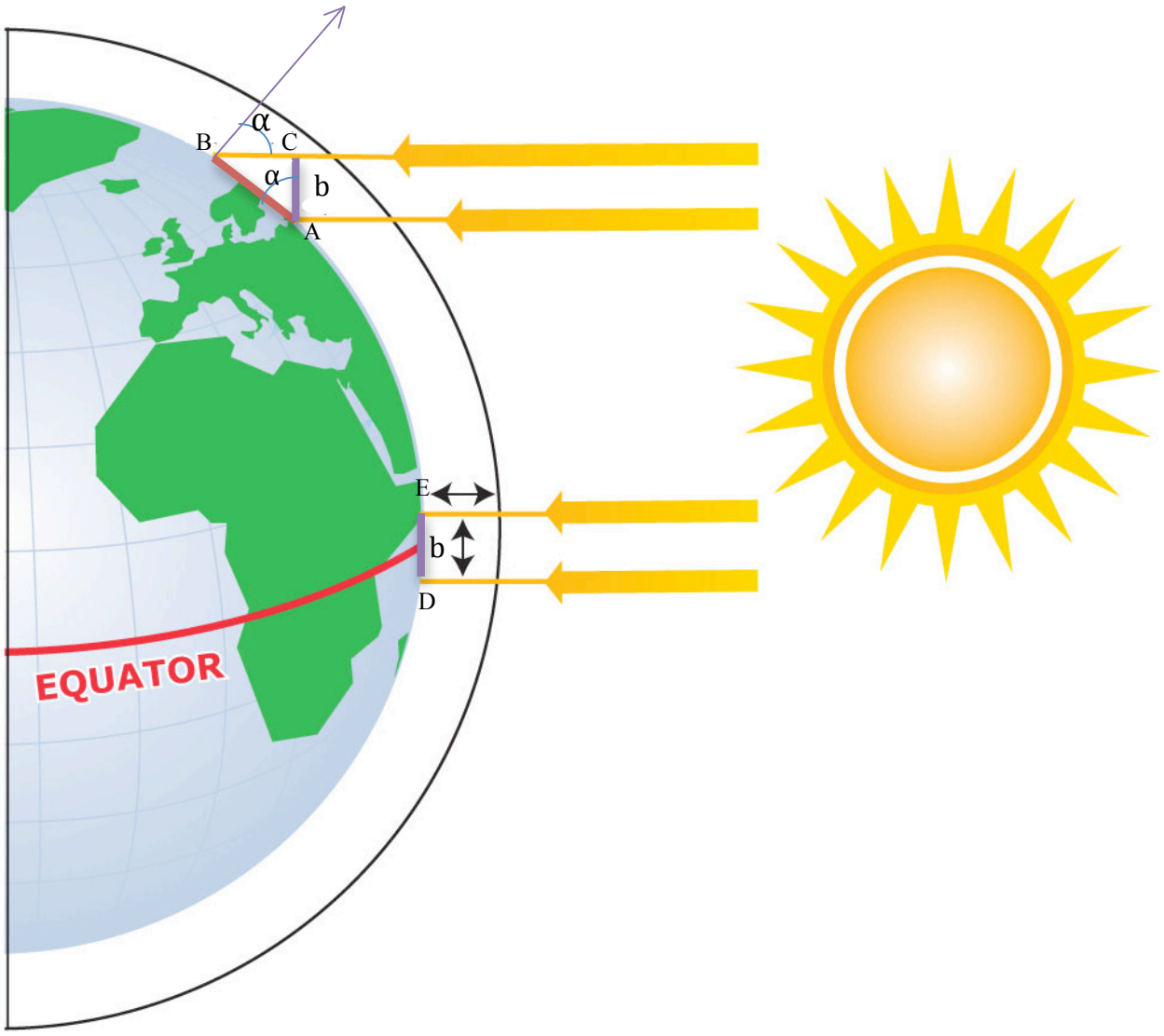
What is your rationale for your recommendation?_____

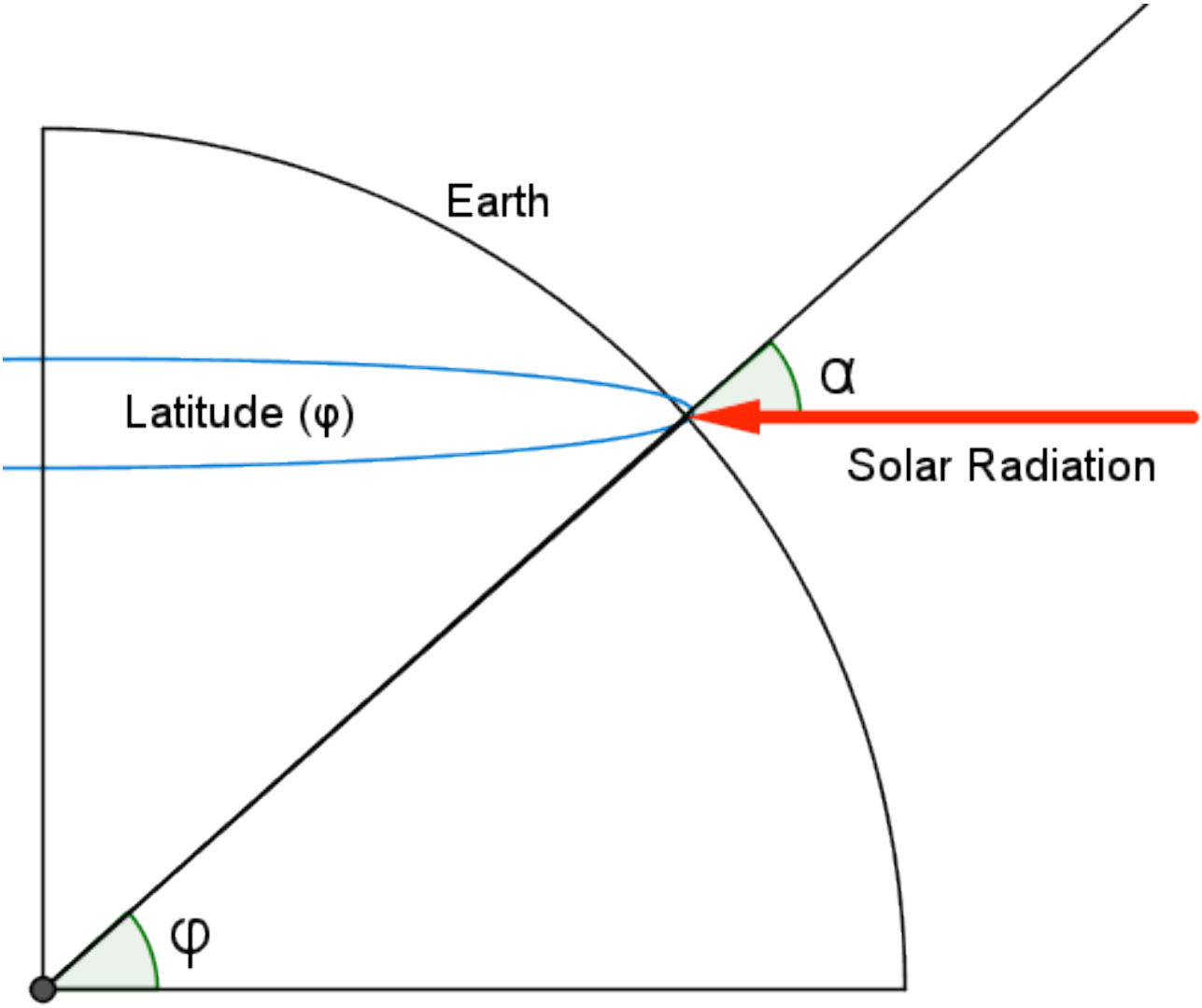
_____

_____

Attachment 4

Visuals




Earth
Latitude (φ)
α
Solar Radiation
φ

Attachment 5

Assessment Worksheet 4

Name_____

1. Write an equation for AB as a function of α and AC _____

2. If the constant solar radiation along b perpendicular to the Earth is S, write an equation for the radiation on the Earth's surface (AB) as a function of incidence angle (α) and latitude (φ).

_____

3. If you were given a data table that listed the average yearly temperatures for cities as you travel from the equator towards a Pole, what trend do you think you would see in the temperatures? What would be the reason for such a trend? _____
_____
_____

4. How does the global warming affect the climate zones distribution?
_____
_____

5. How does the climate change effect on weather, wildlife, glaciers, sea levels and human health?
_____
_____
_____
_____
_____
_____

6. Using curves that were constructed from the experimental data what would be your prediction for the temperature reading after 6 min if a thermometer was placed at the angle of 75°?
_____
_____
_____
_____
_____

7. Natalie rides a horse all the way down a straight slope from the top of a hill that is 23 meters tall. The slope makes a 10 degree angle to the flat ground. How many meters long was the ride?